The Cost of Constraints: Risk Management, Agency Theory and Asset Prices

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Asset Pricing

- Fama and Shiller, two alternative views, our third view investor constraints important, cost tradeoffs.
- Anomalies or Risk Differences:
  - High beta stocks earn less on a risk adjusted basis than low beta stocks
  - Black, Jensen Scholes (1972), Fama-Macbeth (1972), Haugen and Heins (1975)
  - Other measures of uncertainty such as volatility and idiosyncratic risk similar results
  - Lakonishok and Shapiro (1986)

Across markets

- Underperformance of stocks with high idiosyncratic volatility extends to international markets
  - Ang, Hodrick, Xing and Zhang (2009)
- Low beta outperformance of high beta stocks same for global fixed income, commodities and currency markets – across multiple asset classes and geographies
  - Frazzini and Pedersen (2013)

Explanations: Behavioral Finance

- Behavioral explanation:
  - Investor interest in lotteries coupled with limits of arbitrage.
  - Noise traders have behavioral biases that lead them to overpay for higher volatility assets and professional investors have limited ability to supply these securities
  - Baker, Bradley and Wurgler [2011] argue this and claim that low risk return anomalies is among greatest puzzles in finance

Explanations: Leverage constraints

- Leverage constraints and higher cost to borrow and lend lead to a flatter capital market line
  - Black (1972) argues for a “kinked” CAPM
  - Frazzini and Pederson (2013) argue for an “elevated and flattened” CAPM

(Our) Constraints Model

- Investors rationally constrain their active investment managers
  - Explicit monitoring costs are high
  - Hard to evaluate performance
  - Easier for risk management purposes
- Managers want constraints to concentrate on skill and not on risk allocation
- Leads to implicit or explicit tracking error constraints

Models with Tracking Error Constraints

- Does the tracking-error constraint lead to a cost in terms of lost returns (efficient set)?
- Does the constraint provide an alternative explanation of the underperformance of high risk assets? Many constrained investment managers affect returns.
- Do investor constraints play a role in explaining anomalies. We have called these Omega returns in that investors knowingly give up returns.

Model

- Roll (1992) model imposed a tracking error constraint on managers. He allowed managers to increase risk using beta to generate excess returns relative to the benchmark by deviating from benchmark weights.
- Showed that these portfolios are also efficient and on the Markowitz efficient frontier.
- We add to the Roll model

Our Extensions of the Roll model

- A.) Tracking error plus a liquidity constraint, (no active management)
  - Managers must hold cash to meet investor redemptions and to make investments
- B.) Tracking error constraint coupled with active management, an alpha portfolio (zero beta and positive beta)
  - With and without a liquidity constraint
  - Implicit in the developed model is a leverage constraint.

Major Findings

- With a liquidity and tracking error constraint
  - The optimal portfolio is no longer mean-variance efficient.
  - To compensate for liquid cash holdings, which cause both performance drag and tracking error, the manager buys higher returning, higher beta stocks to meet his mandate
  - In other words, the manager adds to holdings of higher risk assets (the tracking-error hedging portfolio is a higher risk portfolio).

Major Findings

- With a tracking error constraint and an zero-beta alpha portfolio, the manager finances his active investments by using low volatility stocks. He holds his higher risk assets.
  - The portfolio is inefficient relative to an unconstrained portfolios
  - The greater is the risk to reward ratio of his active portfolio, the more the manager wants to deviate from the benchmark and must finance more of the active position from lower risk assets holding higher risk assets to mitigate the tracking-error constraint
- Our model has dynamics based on alpha beliefs.

Problem (no alpha)

- $$\min \quad x'\Omega x$$

  subject to:

- $$x'R=G$$

- $$x'1=k<0$$

- Select the weights x, a vector that represents the differences between the weights of the managed portfolio and the benchmark portfolio.

- Ω is the covariance matrix of N assets,

- G is the target outperformance (e.g., 1%) and

- k are the sum of the differential weights caused by the liquidity constraint (e.g. k = -5%)

Solution

$$x = \frac{G}{\mathrm{R}_1 - \mathrm{R}_0}(q_1 - q_0) + \frac{k}{\mathrm{R}_1 - \mathrm{R}_0}(q_0 R_1 - q_1 R_0)$$

- **First term** is Roll's (1992) solution – it is a mean/variance efficient portfolio. It has an expected return = G.
- **Second term** is the deviation from the efficient portfolio due to liquidity constraint – we call this a "hedging portfolio". It has an expected return of zero.
- Solution Logic:
  - First find a portfolio that meets the relative return target of G
  - Then find a hedging portfolio that reduces tracking error

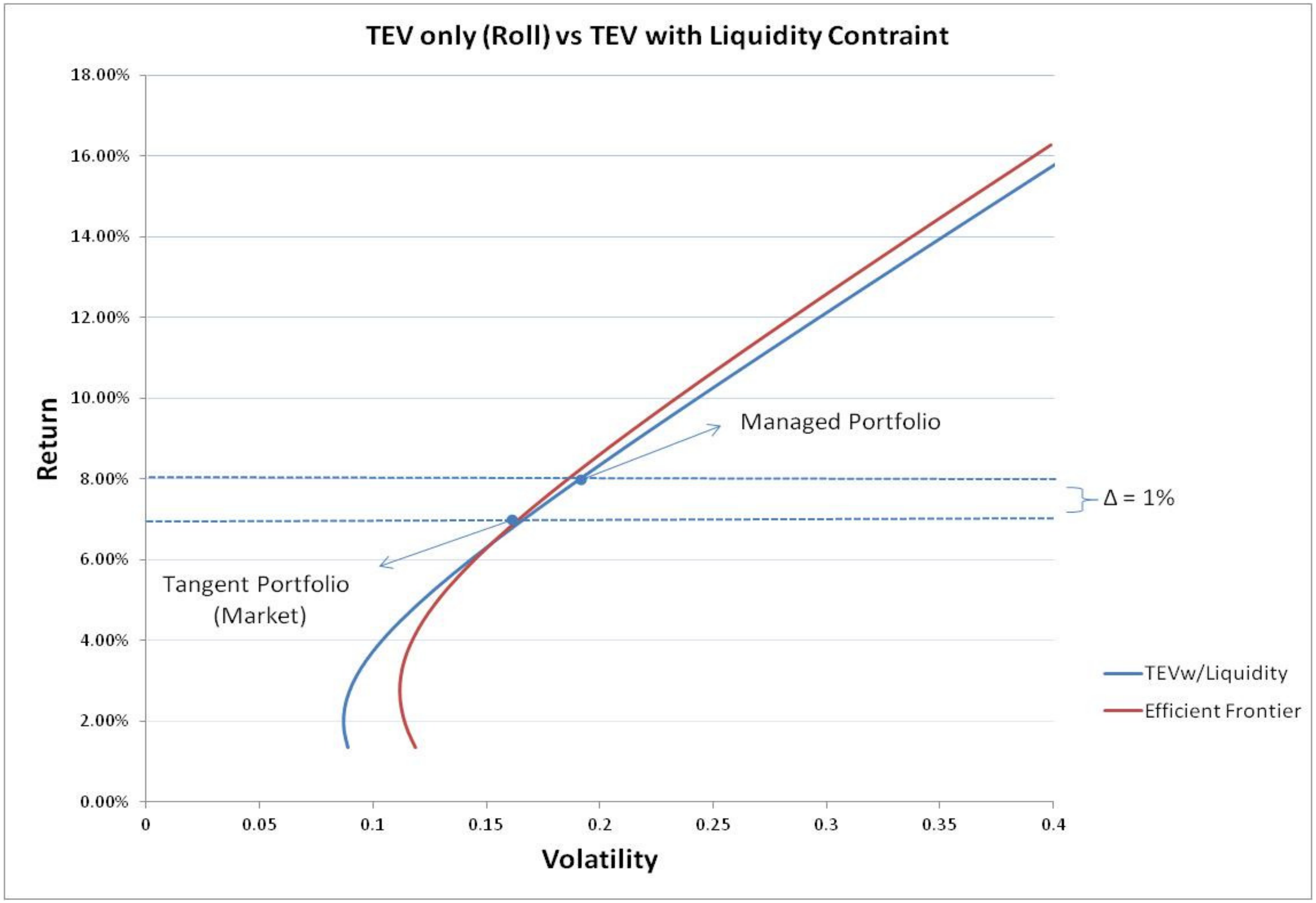
Hedging Portfolios

$$x = \frac{G}{R_1 - R_0}(q_1 - q_0) + \boldsymbol{\frac{k}{R_1 - R_0}(q_0 R_1 - q_1 R_0)}$$

- q_0 and q_1 are special mean-variance efficient portfolios.
  - q_0 = minimum volatility portfolio, q_1 = higher volatility portfolio
- Shorts q_0 (a low volatility portfolio) and buys q_1 (a high volatility portfolio) with net position = k < 0.
- Overweighting higher-volatility counters the tracking error stemming from zero risk cash holdings

<u>Low vol stocks supplied, high vol stocks demanded</u>

Efficient Set And Constrained Portfolio



Parameters: Tracking Error of 5.5%, Liquidity of 5% (k = -5%), G = 1%

Solution with Alpha, Liquidity and Tracking Error

- With alpha present, an additional amount of q_0 , the low volatility portfolio, is shorted to finance exposure to alpha
- There is also an interaction term between the liquidity and the alpha portfolio (both are “zero beta” assets”)
- Without the liquidity constraint the alpha and tracking error constraints interact like the liquidity constraint
- Dynamics of solution. Demand for high volatility stocks depends on alpha strength

Alpha Portfolio and Liquidity

- Portfolio (zero-beta) with an expected alpha to variance ratio of $\frac{\propto}{\sigma_\alpha^2}$. Solution:

- $$x = \frac{\begin{bmatrix} G(q_1 - q_0) + k(q_0 R_1 - q_1 R_0) + G\left(\frac{q_1 \sigma_0^2}{\sigma_\alpha^2} - \frac{q_0 \alpha \sigma_0^2}{\sigma_\alpha^2 R_0}\right) + k\alpha\left(\frac{q_0 \alpha \sigma_0^2}{\sigma_\alpha^2 R_0} - \frac{q_1 \sigma_0^2}{\sigma_\alpha^2}\right) \\ G\left(\frac{\alpha \sigma_0^2}{\sigma_\alpha^2 R_0} - \frac{\sigma_0^2}{\sigma_\alpha^2}\right) + \frac{k \sigma_0^2}{\sigma_\alpha^2}(R_1 - \alpha) \end{bmatrix}}{\left(R_1 - R_0 + \frac{R_1 \sigma_0^2}{\sigma_\alpha^2} + \frac{\alpha^2 \sigma_0^2}{\sigma_\alpha^2 R_0} - \frac{2\alpha \sigma_0^2}{\sigma_\alpha^2}\right)} \quad \} \text{ Alpha position}$$

- where the last row of x (in the numerator) represents the manager's allocation to the alpha portfolio and k < 0.
  - Numerator is a vector of length N+1 assets. (First row is of length N of non-alpha assets.)
  - Denominator normalizes weights so sum equals k. (k = 0 both benchmark and optimal portfolio weights sum to one.)

- Alpha portfolio, like cash, leads to tracking error. Balance the expected gains of allocating more risk to alpha against increases in tracking error.

Alpha and No Liquidity Constraint, k=0

- Simplified solution becomes:

$$x = \frac{\begin{bmatrix} G(q_1 - q_0) + G\left(\frac{q_1\sigma_0^2}{\sigma_\alpha^2} - \frac{q_0\alpha\sigma_0^2}{\sigma_\alpha^2 R_0}\right) \\ G\left(\frac{\alpha\sigma_0^2}{\sigma_\alpha^2 R_0} - \frac{\sigma_0^2}{\sigma_\alpha^2}\right) \end{bmatrix}}{(R_1 - R_0 + \frac{R_1\sigma_0^2}{\sigma_\alpha^2} + \frac{\alpha^2\sigma_1^2}{\sigma_\alpha^2 R_1} - \frac{2\alpha\sigma_0^2}{\sigma_\alpha^2})}$$

Alpha position

-
- Position in alpha funded by selling q_0 (low volatility stocks) → tradeoff between q_0 and α. The amount allocated to alpha is increasing with the risk adjusted return of the alpha vs. q_0,
- $$\frac{\alpha\sigma_0^2}{\sigma_\alpha^2 R_0} = \frac{\alpha/\sigma_\alpha^2}{R_0/\sigma_0^2}.$$
- More $G\frac{\sigma_0^2}{\sigma_\alpha^2}$ of q_1 is bought to compensate for low risk (zero beta) of alpha.

Joint Alpha And Liquidity Constraint

- An interaction between both cash and zero-beta alpha portfolio given tracking-error constraints.
- A cash constraint, k<0, and alpha compounds the underweight in low volatility stocks and the overweight in higher volatility stocks. The impact is multiplicative in $k\alpha$, (rightmost term of the top row of EQ. 3).
- $$k\alpha\left(\frac{q_0\alpha\sigma_0^2}{\sigma_\alpha^2 R_0} - \frac{q_1\sigma_0^2}{\sigma_\alpha^2}\right)$$
- Zero-beta alpha can not hedge the tracking error of holding cash (also zero beta). The liquidity constraint to hold cash leads to a smaller allocation to alpha versus no requirement to hold cash (as long as alpha is sufficiently small).
- If α < R_1, the alpha position is reduced by $\frac{k\sigma_0^2}{\sigma_\alpha^2}(R_1 - \alpha)$. Otherwise, helps meet the outperformance return target offering a higher expected return than the alternative q_1 portfolio despite its inability to hedge the tracking error associated with holding cash.

Alpha, Tracking Error and Leverage

- $K > 0$, allows for leverage

The Interaction of Alpha, Tracking Error and Leverage (K > 0)

- If leverage allowed, at the optimal amount of leverage, mangers still underweight high volatility securities.

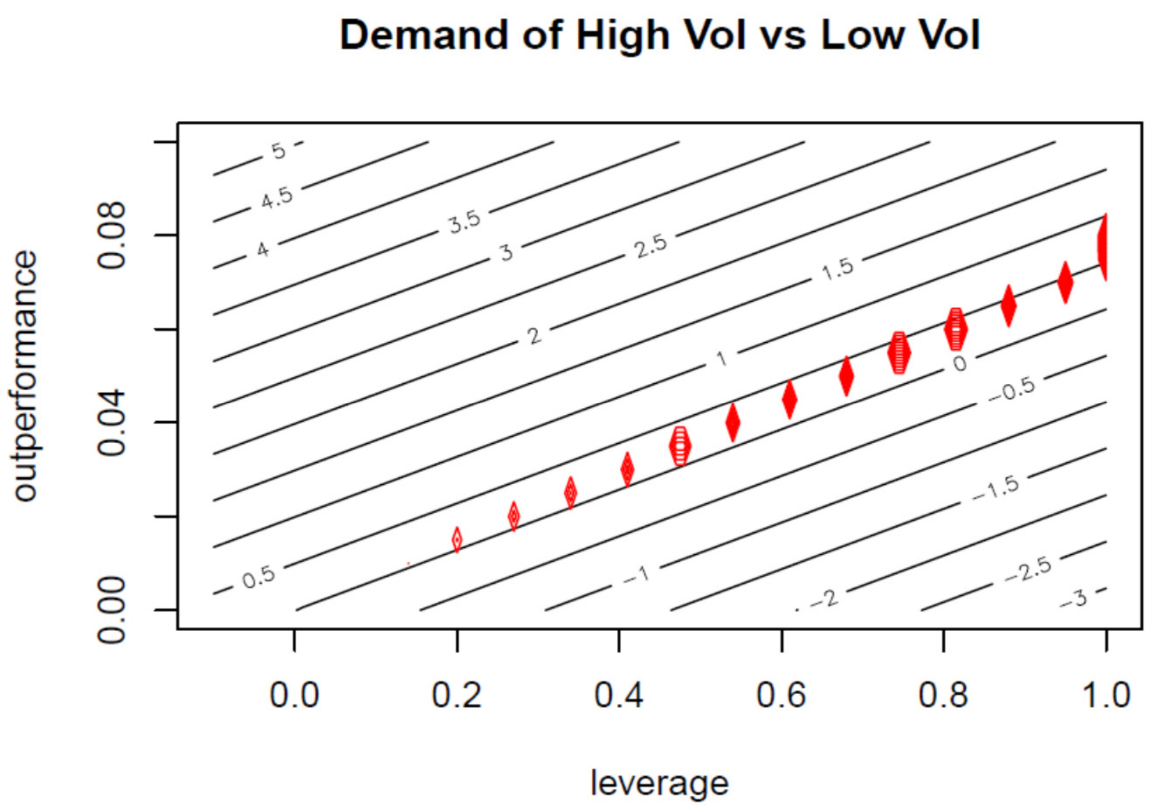
- Position in q_1 (high vol) = $G\left(\frac{\sigma_\alpha^2+\sigma_0^2}{\sigma_\alpha^2}\right) - k\left(\frac{R_0\sigma_\alpha^2+\alpha\sigma_0^2}{\sigma_\alpha^2}\right)$
  - Increasing in G and decreasing in k (leverage) depending on alpha

- Position in q_0 (low vol) = $k\left(\frac{R_0\sigma_\alpha^2+\alpha^2\sigma_0^2}{R_0\sigma_\alpha^2}\right) - G\left(\frac{R_0\sigma_\alpha^2+\alpha\sigma_0^2}{R_0\sigma_\alpha^2}\right)$
  - Decreasing in G and increasing in k(leverage) depending on alpha

- Compared to k = 0, leveraged portfolio less exposure to alpha and more exposure to q_0 and less q_1 for reasonable values of alpha.

High versus Low Vol Stocks

- Relative optimal excess demand (optimal versus benchmark holdings) of high volatility versus low volatility stocks indicated as all red dots for different outperformance and leverage above the zero contour line.



Understanding Leverage

- $$x = \frac{\left[\frac{G(q_1-q_0)+k(q_0R_1-q_1R_0)+G\left(\frac{q_1\sigma_0^2}{\sigma_\alpha^2}-\frac{q_0\alpha\sigma_0^2}{\sigma_\alpha^2R_0}\right)+k\alpha\left(\frac{q_0\alpha\sigma_0^2}{\sigma_\alpha^2R_0}-\frac{q_1\sigma_0^2}{\sigma_\alpha^2}\right)}{G\left(\frac{\alpha\sigma_0^2}{\sigma_\alpha^2R_0}-\frac{\sigma_0^2}{\sigma_\alpha^2}\right)+\frac{k\sigma_0^2}{\sigma_\alpha^2}(R_1-\alpha)}\right]}{(R_1-R_0+\frac{R_1\sigma_0^2}{\sigma_\alpha^2}+\frac{\alpha^2\sigma_0^2}{\sigma_\alpha^2R_0}-\frac{2\alpha\sigma_0^2}{\sigma_\alpha^2})}$$
- (1) Outperformance target, G, is achieved by assuming k = 0. (long q_1, short, q_0, long α)
- (2) Tracking error reduced when k > 0. The "k" portfolio is zero expected return and goes opposite of (1), (long q_0, short q1 and long/short α depending on how much α is held in (1) which is a function of α return)
- If $\alpha > R_1$ the solution to (1) hold a lot of alpha → contributes a lot to tracking error, in (2), α is shorted. Why would investors want less alpha and pay for more unskilled returns? Investors could constrain managers to k = 0, who claim large excess returns, $\alpha > R_1$. Tolerating tracking error to prevent "cheating".
- If $\alpha < R_1$, k> 0, will generate additional α. Tracking error portfolio goes short q_1 and buys q_0, which has a cost proportional to α - R_1. But, this cost is exactly offset by the gain in α. Again, investors might restrict k = 0.
- There is also a tradeoff between gain from leveraging both α and q_0. More gain from leveraging α, relative to q_0, if α is less than $R_1/2$, otherwise not.

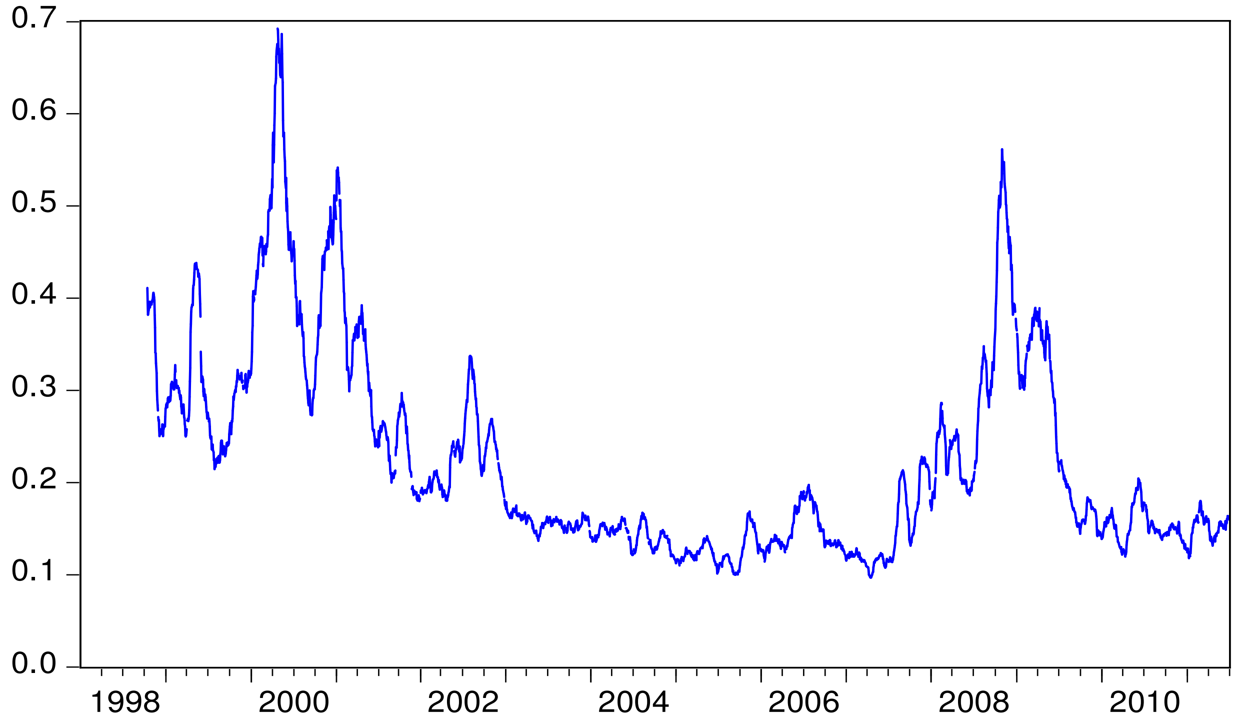
Empirical Results

$$x = \frac{\left[\begin{array}{c} G(q_1 - q_0) + G\left(\frac{q_1\sigma_0^2}{\sigma_\alpha^2} - \frac{q_0\alpha\sigma_0^2}{\sigma_\alpha^2 R_0}\right) \\ G\left(\frac{\alpha\sigma_0^2}{\sigma_\alpha^2 R_0} - \frac{\sigma_0^2}{\sigma_\alpha^2}\right) \end{array}\right]}{\left(R_1 - R_0 + \frac{R_1\sigma_0^2}{\sigma_\alpha^2} + \frac{\alpha^2\sigma_1^2}{\sigma_\alpha^2 R_1} - \frac{2\alpha\sigma_0^2}{\sigma_\alpha^2}\right)}$$

- Used a sample of 95 active mutual fund mangers (monthly data from end of 1999 through June 2013.)
- Benchmark was S&P 500.
- Tracking error plus/minus 10% range.

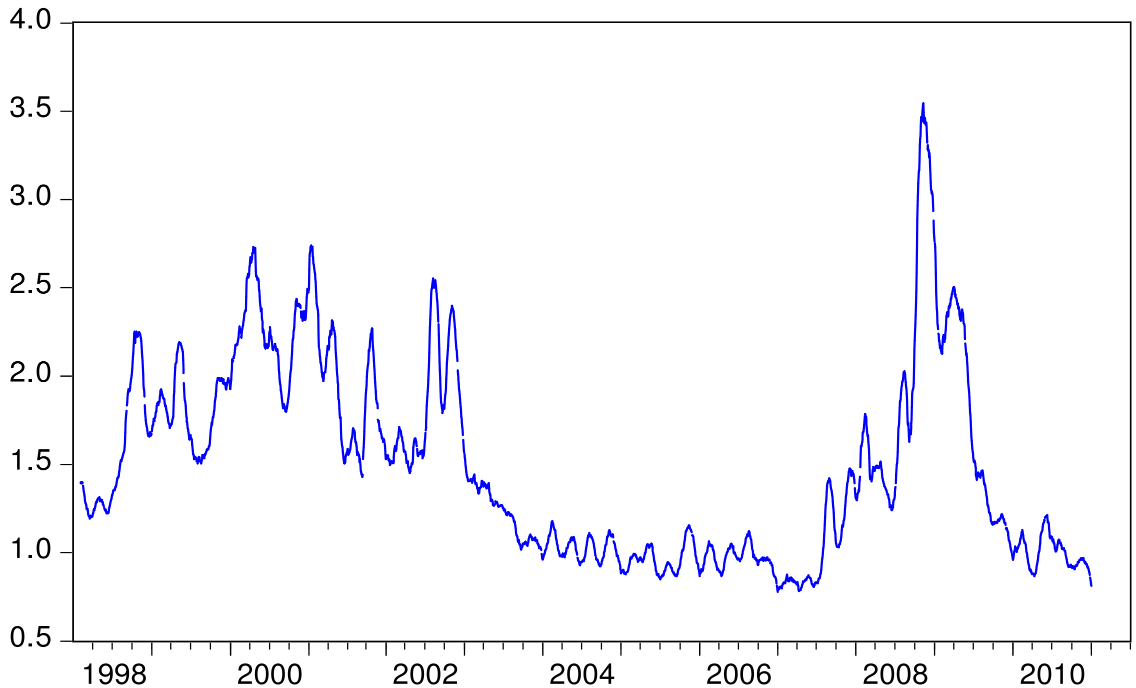
30 day moving average of median mutual fund absolute tracking error in percent

- Tracking error from fund benchmarks. Affected by:
  - Differences in holdings
  - Single stock correlations



30 day moving average of the average of S&P500 stocks' daily absolute beta-adjusted deviation from the S&P500 in percent

- This is a measure of single stock cross-sectional correlation.
  - High value in graph below implies cross-sectional correlation low.



Regression mutual fund tracking error on single stock correlation

- Results:

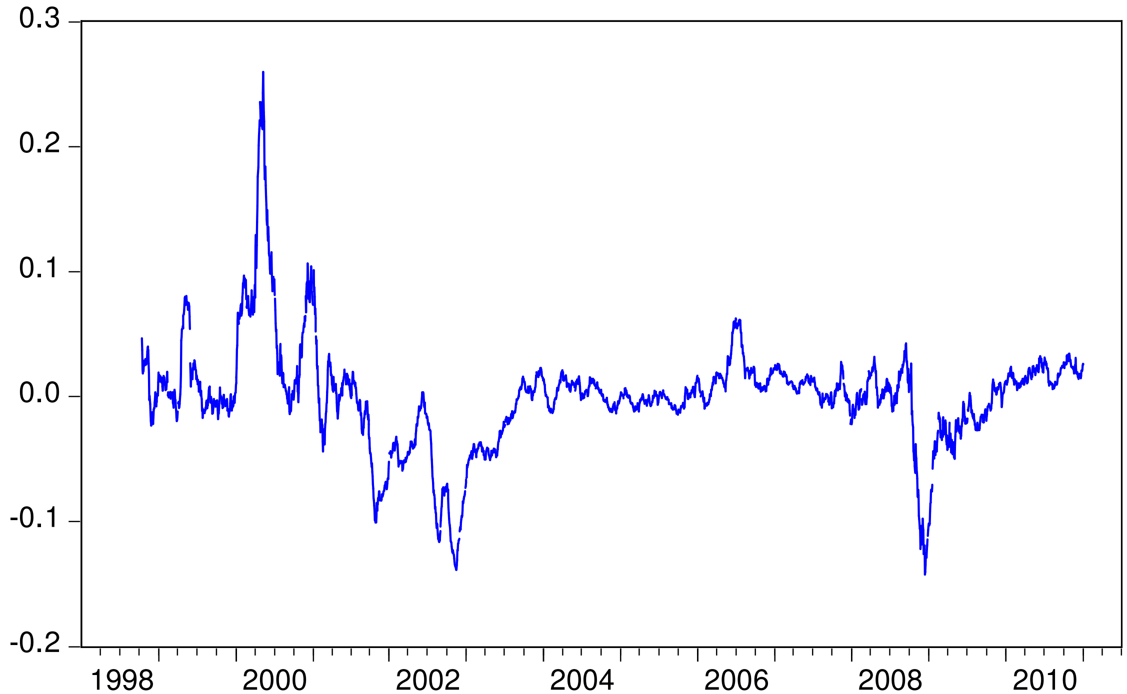
Regression 3 - Tracking Error

R Square	0.61
Observations	3104

	Coefficient	SE	t Stat	P-value
Intercept	-0.001	0.000	-12.53	0.000
SPX_ABS_IDIO	0.189	0.003	70.29	0.000

- Residual represents tracking error stemming from differences in holdings, not from changes in single stock correlation.

30 day moving average of residuals – Explicit measure of tracking error



Dynamics: Tracking Error Changes and Low vs. High Volatility Anomaly

Major Changes in Target Tracking Error

Date		Tracking Error Index			Low Volatility - High Volatility
Start		*Start*	*End*	*Change*	*Expected Performance*
Sep-98	Apr-00	(0.2%)	5.8%	6.0%	Underperform
Apr-00	Oct-02	5.8%	(3.3%)	(9.1%)	Outperform
Oct-02	Nov-03	(3.3%)	0.5%	3.8%	Underperform
Mar-08	Nov-08	0.7%	(3.1%)	(3.7%)	Outperform
Nov-08	May-10	(3.1%)	0.5%	3.5%	Underperform

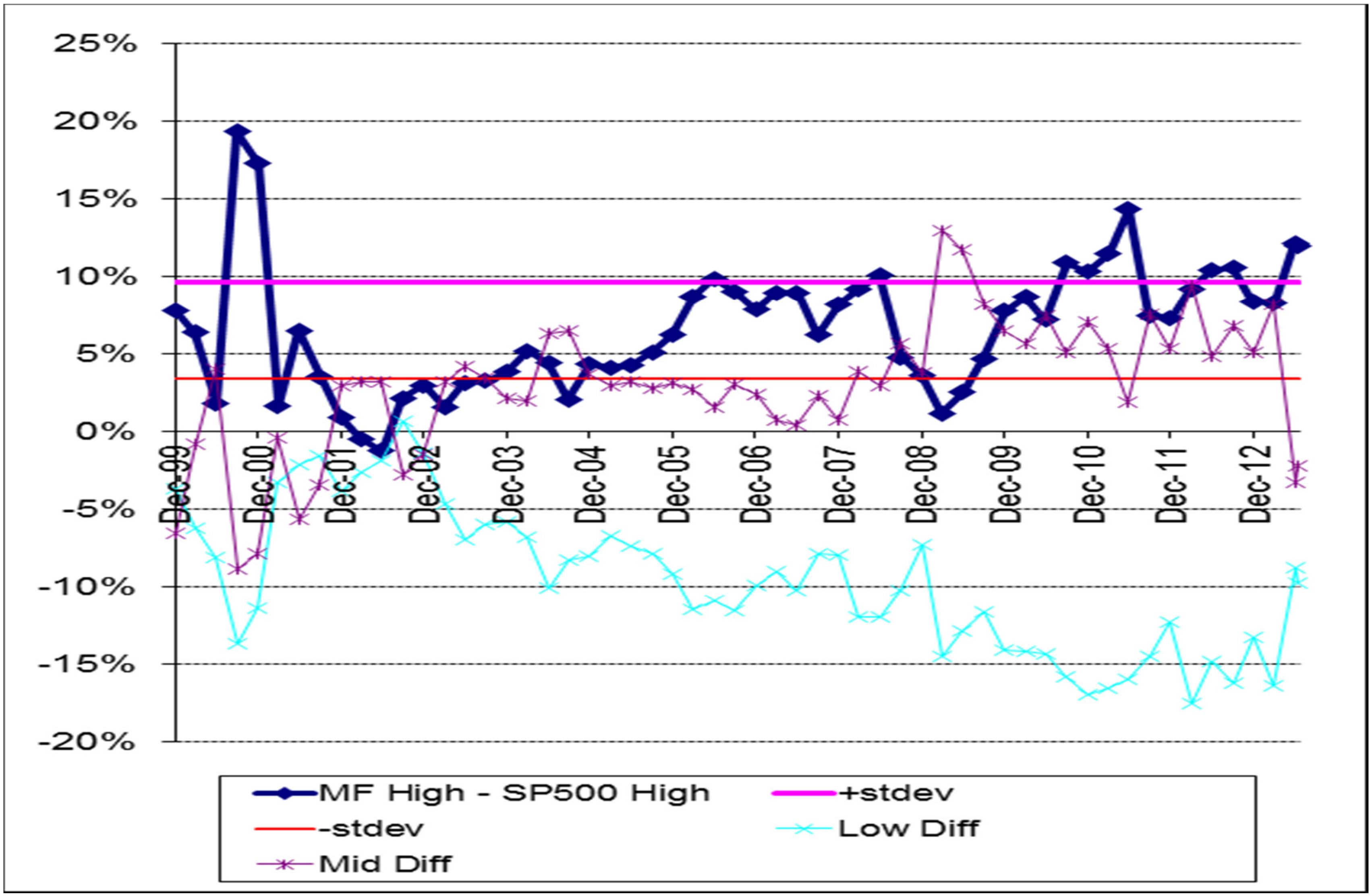
Major Changes in Target Tracking Error

Date			Realized Factor Performance					
Start	**End**	**Low Volatility - High Volatility Expected Performance**	*Average*	*nVolatility*	*BBeta*	*BVol*	*BTRrisk*	*BSRisk*
Sep-98	Apr-00	Underperform	(2.6%)	(14.8%)	(2.7%)	3.2%	(7.5%)	8.7%
Apr-00	Oct-02	Outperform	37.7%	9.0%	41.3%	26.3%	57.1%	54.6%
Oct-02	Nov-03	Underperform	(16.2%)	(11.2%)	(10.8%)	(15.9%)	(22.0%)	(21.0%)
Mar-08	Nov-08	Outperform	13.2%	7.4%	15.7%	11.6%	15.0%	16.2%
Nov-08	May-10	Underperform	(39.5%)	(13.6%)	(51.0%)	(42.5%)	(47.8%)	(42.4%)

What is the Empirical Cost of Constraints?

- We (1) measure mutual fund manager returns and (2) control those returns for the tracking error constraint.
- The differences in alpha and beta(s) between (1) and (2) give our estimate of the implicit cost of the tracking error constraint.
- Form an aggregate mutual fund portfolio by using weights at the end of each quarter of all the equity holdings (aggregated of 95 funds)

Same MF Holdings vs SPX, Grouped by Volatility



Summary Statistics

- Low volatility stocks underweight in active mutual funds.

		Market Value Of Stocks in Respective Portfolios				
		S&P Groups			Mutual Fund Groups	
		Mean	Std Dev		Mean	Std Dev
Low Volatility		66.04%	6.01%		56.17%	6.04%
Mean Difference		-9.87%	4.52%			
	t-stat	-16.31				

- Mutual funds outperform in all volatility groups.

Intercept of differences in returns of portfolio groups from S&P group					
	Regressed on S&P Portfolio Returns.				
	Port8 -10	Port 4-7	Port 1-3		
	0.55%	0.44%	0.54%		
	t = 1.39	t = .83	t = 1.53		
Note: All slopes insignificantly different from zero.					

Aggregate Portfolio on S&P Returns (Quarterly Data)

R Square	0.94
Observations	56

	Coefficient	SE	t Stat	P-value
Intercept	0.365	0.313	1.17	0.249
S&P	1.073	0.037	29.04	0.000

Aggregate on S&P and overweight High volatility

R Square	0.94
Observations	56

	Coefficients	SE	t Stat	P-value
Intercept	1.290	0.601	2.14	0.037
S&P 500 Returns	1.073	0.036	29.65	0.000
8 to 10 Holding	-0.122	0.068	-1.79	0.079

Intercept increases from 0.36% per quarter to 1.29% per quarter.
→ Cost of the tracking error constraint is 0.93% per quarter!

Note: 8 to 10 Holding same as blue line in slide 29 and represented in regression in number, i.e. 10% -> 10

Conclusion

- Active managers do take tracking error and the level changes with uncertainty of alpha.
- They do overweight higher volatility assets
- Managers do have significant skill but pay a high cost with lost returns. Tradeoff between explicit monitoring costs and implicit costs through lower returns.
- Constraints model has rich implications for asset management and explanation of anomalies.