Forecast Risk Bias in Optimized Portfolios

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Portfolio Construction

- Portfolio construction is the process of determining asset weights that best represent return and risk trade-off.
Portfolio Construction

Optimization
Maximizes a utility function, $U(P)$.
$U(P) = \alpha - \lambda \sigma_p^2 - TC - \ldots$

- What if there are errors in the inputs?
Errors in Expected Return Estimates

- A wealth of research over the years has dealt with errors in expected return estimates
  - The problem was first described by Barry (1974), Michaud (1989), and Jorion (1992)
- Since then, proposed frameworks to deal with the problem include:
  - Black-Litterman
  - Robust optimization w/ alpha error estimates
  - Bayesian methods / Shrinkage
- But note...
  - Kritzman (2006) argues that the return distribution of the presumed optimal portfolio is actually similar to the distribution of the truly optimal portfolio. Thus, mean-variance optimizers usually turn out to be more robust to small input errors than conventional wisdom assumes
Errors in Risk Model Estimates

- Covariance matrices are also subject to estimation (or sampling) error:
- As with expected returns, any sample covariance matrix contains estimation error
  - Especially when the number of stocks >> the number of time periods for observed returns
- “Error maximization” (Michaud, 1989)
  - When the sample covariance matrix is an input to a mean-variance optimizer, it will result in ‘extreme’ and under-diversified portfolios
Errors in Risk Model Estimates

- Some solutions have been proposed

  - Not based upon an improved estimator of the covariance matrix
  - From artificial return data resampled from the observed data, covariance matrices are sampled many times and fed into the mean-variance optimizer.
  - The optimal portfolios which result are then averaged.

- Ledoit and Wolf (2004):
  - Propose an improved estimator of the covariance matrix based on “shrinkage.”
  - Shrinkage pulls the most extreme coefficients towards more central values
  - Specifically finds an optimal linear combination of the sample covariance matrix and a highly structured estimator, which assumes that the correlation between the returns of any two stocks is always the same
Sampling Error

- Sampling error: Covariance matrix is based on a limited number of observations.
- Estimating $\Sigma$ for $n$ assets over $T$ time periods ($T>n$)

\[
\frac{\text{Estimated Variance}}{\text{True Variance}} = \frac{E\left(\hat{h}^*\hat{\Sigma}h^*ight)}{E\left(\hat{h}^*\Sigma h^*ight)} \approx \left(1 - \frac{n}{T}\right)^2
\]

- Ratios below one represent underforecasting bias thus risk forecasts of optimized portfolios are biased low.
Sampling Error

- If the universe consists of 100 assets and we construct the sample covariance matrix from weekly returns over 5 years of history, the forecast variance of an optimized portfolio is roughly 37% of the true variance.
- If we expand the universe to 200 stocks, the forecast is only 5% of the true variance — a 95% underestimation!
Factor Model – Structure Helps

- Assume that the factor structure is known (i.e., there is no model error) and exposures to these factors are known

$$\Sigma = X F X^T + \Delta$$

- We can show that the relevant ratio is now $\frac{k}{T}$ not $\frac{n}{T}$

- Sampling mainly affects $F$, a $k \times k$ matrix, which has much fewer dimensions than $n \times n$

- With five years of weekly returns, the average bias is less than 3%, regardless of the number of assets

- Moreover, the greater proportion of specific risk in the portfolio, the less severe the effects of the errors
Simulations: How Bad is the Bias?

- Start with the Barra US Equity Short Term Model (USE3S) as of March 2008
  - 68 factors in the model
  - Assume this is the “true” risk model
- We build two types of risk models over many simulations:
  - In each simulation, we generate histories of factor and specific returns (Z and w are multivariate standard normal):
    \[ u = \Delta^{1/2} Z \quad f = F^{1/2} w \]
  - Asset-by-asset covariance matrix: In each simulation run, we build a covariance matrix from a history of 200 periods of returns
  - Factor-based covariance matrix: In each simulation, we build the factor covariance matrix and specific risk matrix separately; we assume that the asset factor exposures are known and need not be estimated
Simulations: How Bad is the Bias?

- We run two types of unconstrained, active optimizations:
  - Stock selection
    - Alphas are unrelated to the model factors
  - Factor tilt
    - Alphas are a randomly weighted combination of three USE3 style factors
    - The weights change with each simulation run
- **Universe/Benchmark = the 100 largest capitalization companies in the MSCI US Prime Market 750 Index**
Simulation Results

- Simulation results for 100 assets:

<table>
<thead>
<tr>
<th>Risk Model</th>
<th>Risk</th>
<th>Stock Selection</th>
<th>Factor Tilt</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>Forecast over Truth (%)</td>
<td>Ratio of Component to Active Variance (%)</td>
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<tr>
<td>Historical Asset</td>
<td>Active Variance</td>
<td>24.4</td>
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</tr>
<tr>
<td>Factor Based</td>
<td>Active Variance</td>
<td>96.7</td>
<td>100.0</td>
</tr>
<tr>
<td></td>
<td>Factor</td>
<td>83.7</td>
<td>11.4</td>
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<tr>
<td></td>
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<tr>
<td></td>
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- Simulation results for 750 assets:

<table>
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<td>Forecast over Truth (%)</td>
<td>Ratio of Component to Active Variance (%)</td>
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<td>Factor Based</td>
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<td>97.2</td>
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Adding Constraints

- So far, we have been looking at unconstrained optimizations
- What if there are constraints?
  - Conventional wisdom: constraints act to limit the error-maximizing behavior of optimization
- Consider the case in which a manager constrains $J$ characteristics of an (active) portfolio with $N$ assets to be exactly zero by imposing the constraints:

  \[ Ah = 0 \]
Adding Constraints

- These equality constraints effectively reduce the number of variables in the problem, since they enable us to write the optimization problem in terms of \( N-J \) assets, rather than \( N \), as follows:

\[
Ah = A_{Jx} h_{Jx1} + A_{Jx(N-J)} h_{(N-J)x1} = 0
\]

\[
h_{Jx1} = -A^{-1}_{Jx} A_{Jx(N-J)} h_{(N-J)x1}
\]

- In turn, this generally reduces the forecasting bias
  - Since factor risk is \( w' F w \)
  - When we constrain a factor, say \( i \), we set \( w_i = 0 \).
  - Effectively drops a variable from the problem
  - Moreover, drops it from the factor risk, which is the principal source of forecasting bias
Adding Constraints: Simulations

- **Rerun simulations:**
  - Case 1: Constrain all factor exposures to be zero, except for the three factors comprising the alpha
  - Case 2: Add long-only constraint

### Forecast over Truth (%)

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<td>Long only Active Risk 3%</td>
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<td>80.3</td>
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Conclusion

- Due to noise in the covariance matrix, portfolio optimization tends to produce portfolios for which the risk forecasts are underestimates of the true risk
- In the case in which the asset returns have a factor structure, using a factor-based covariance matrix mitigates the risk forecast bias significantly
- Furthermore, our analysis reveals that the bias in factor model risk forecasts may be significantly less than earlier estimates would suggest
- Finally, we discuss briefly how constraints mitigate the forecast bias
References

- Presentation is based on the paper: "Forecast Risk Bias in Optimized Portfolios", MSCI Barra Research Insight, October 2009 – Bender, Lee, Stefek, Yao
- Additional citations:
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