MSCI A Clear View of Risk and Return

Barra

Forecast Risk Bias in Optimized Portfolios

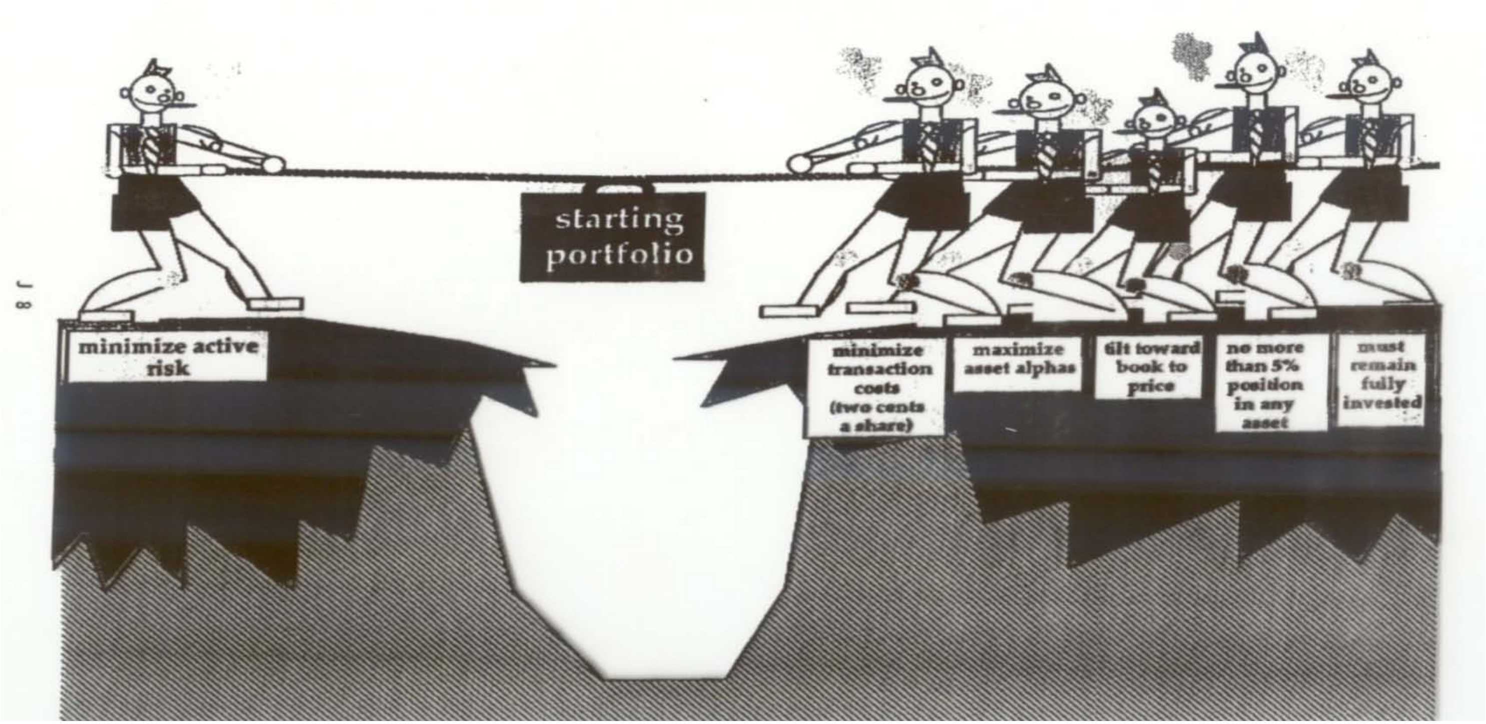
March 2011

Presented to Qwafafew, Denver Chapter

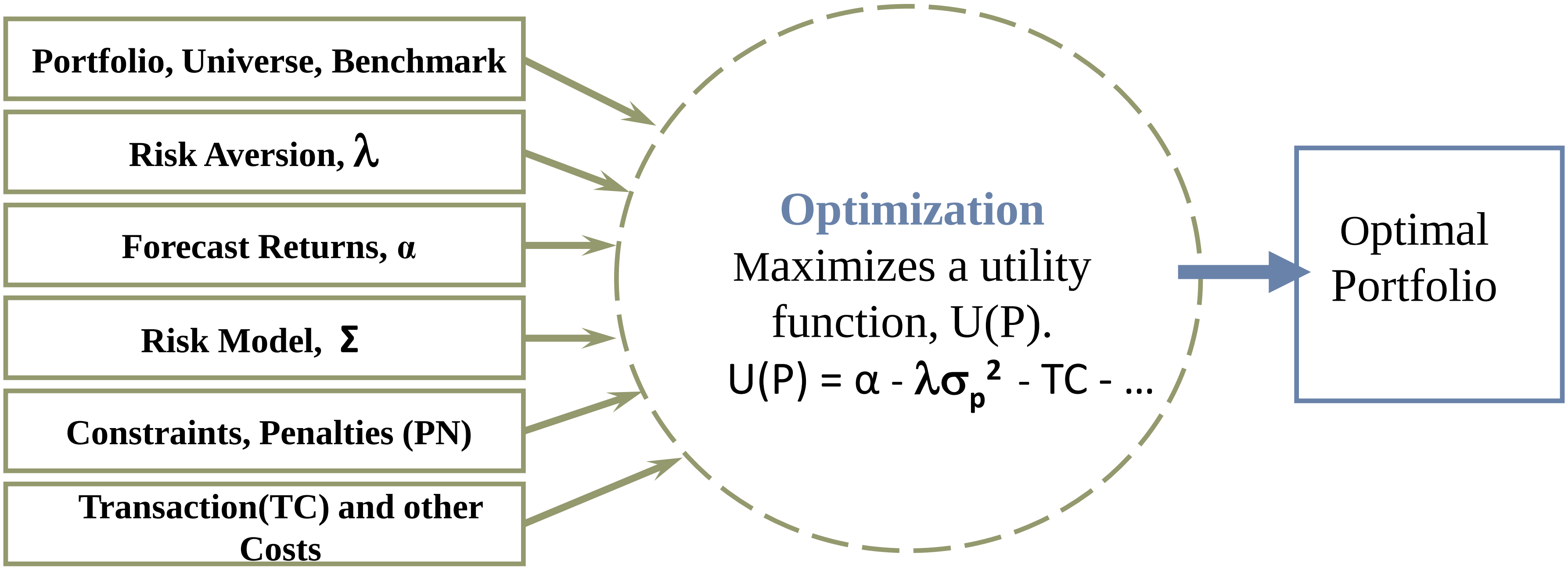
Jenn Bender, Jyh-Huei Lee, Dan Stefek, Jay Yao

Portfolio Construction

- Portfolio construction is the process of determining asset weights that best represent return and risk trade-off



Portfolio Construction



- What if there are errors in the inputs?

Errors in Expected Return Estimates

- A wealth of research over the years has dealt with errors in expected return estimates
  - The problem was first described by Barry (1974), Michaud (1989), and Jorion (1992)
- Since then, proposed frameworks to deal with the problem include:
  - Black-Litterman
  - Robust optimization w/ alpha error estimates
  - Bayesian methods / Shrinkage
- But note…
  - Kritzman (2006) argues that the return distribution of the presumed optimal portfolio is actually similar to the distribution of the truly optimal portfolio. Thus, mean-variance optimizers usually turn out to be more robust to small input errors than conventional wisdom assumes

Errors in Risk Model Estimates

- Covariance matrices are also subject to estimation (or sampling) error:
- As with expected returns, any sample covariance matrix contains estimation error
  - Especially when the number of stocks >> the number of time periods for observed returns
- “Error maximization” (Michaud, 1989)
  - When the sample covariance matrix is an input to a mean-variance optimizer, it will result in ‘extreme’ and under-diversified portfolios

Errors in Risk Model Estimates

- Some solutions have been proposed
- Michaud (1998) “Resampling”:
  - Not based upon an improved estimator of the covariance matrix
  - From artificial return data resampled from the observed data, covariance matrices are sampled many times and fed into the mean-variance optimizer.
  - The optimal portfolios which result are then averaged.
- Ledoit and Wolf (2004):
  - Propose an improved estimator of the covariance matrix based on “shrinkage.”
  - Shrinkage pulls the most extreme coefficients towards more central values
  - Specifically finds an optimal linear combination of the sample covariance matrix and a highly structured estimator, which assumes that the correlation between the returns of any two stocks is always the same

Sampling Error

- Sampling error: Covariance matrix is based on a limited number of observations
- Estimating Σ for n assets over T time periods (T>n)

$$\frac{\textit{Estimated Variance}}{\textit{True Variance}} = \frac{E\left(\hat{h}^{*\prime}\hat{\Sigma}\hat{h}^{*}\right)}{E\left(\hat{h}^{*\prime}\Sigma\hat{h}^{*}\right)} \approx \left(1 - \frac{n}{T}\right)^{2}$$

- Ratios below one represent underforecasting bias thus risk forecasts of optimized portfolios are biased low

Sampling Error

- If the universe consists of 100 assets and we construct the sample covariance matrix from weekly returns over 5 years of history, the forecast variance of an optimized portfolio is roughly 37% of the true variance
- If we expand the universe to 200 stocks, the forecast is only 5% of the true variance — a 95% underestimation!

Factor Model – Structure Helps

- Assume that the factor structure is known (i.e., there is no model error) and exposures to these factors are known

$$\Sigma = \underbrace{X\ F\ X^{T}}_{\text{Factor risk}} + \underbrace{\Delta}_{\text{Idiosyncratic risk}}$$

- We can show that the relevant ratio is now $\frac{k}{T}$ not $\frac{n}{T}$
- Sampling mainly affects F, a $k \times k$ matrix, which has much fewer dimensions than *n x n*
- With five years of weekly returns, the average bias is less than 3%, <u>regardless of the number of assets</u>
- Moreover, the greater proportion of specific risk in the portfolio, the less severe the effects of the errors

Simulations: How Bad is the Bias?

- Start with the Barra US Equity Short Term Model (USE3S) as of March 2008
  - 68 factors in the model
  - Assume this is the “true” risk model
- We build two types of risk models over many simulations:
  - In each simulation, we generate histories of factor and specific returns (Z and w are multivariate standard normal):

$$u = \Delta^{1/2} z \qquad f = F^{1/2} w$$

  - Asset-by-asset covariance matrix: In each simulation run, we build a covariance matrix from a history of 200 periods of returns
  - Factor-based covariance matrix: In each simulation, we build the factor covariance matrix and specific risk matrix separately; we assume that the asset factor exposures are known and need not be estimated

Simulations: How Bad is the Bias?

- We run two types of unconstrained, active optimizations:
  - Stock selection
    - Alphas are unrelated to the model factors
  - Factor tilt
    - Alphas are a randomly weighted combination of three USE3 style factors
    - The weights change with each simulation run
- Universe/Benchmark = the 100 largest capitalization companies in the MSCI US Prime Market 750 Index

Simulation Results

- Simulation results for 100 assets:

Risk Model	Risk	Stock Selection		Factor Tilt	
		Forecast over Truth (%)	**Ratio of Component to Active Variance (%)**	**Forecast over Truth (%)**	**Ratio of Component to Active Variance (%)**
Historical Asset	**Active Variance**	24.4	--	24.5	--
Factor Based	**Active Variance**	96.7	100.0	92.7	100.0
	Factor	83.7	11.4	83.5	37.2
	Specific	98.3	88.6	98.1	62.8

Simulation Results

- Simulation results for 100 assets:

Risk Model	Risk	Stock Selection		Factor Tilt	
		Forecast over Truth (%)	**Ratio of Component to Active Variance (%)**	**Forecast over Truth (%)**	**Ratio of Component to Active Variance (%)**
Historical Asset	**Active Variance**	24.4	--	24.5	--
Factor Based	**Active Variance**	96.7	100.0	92.7	100.0
	Factor	83.7	11.4	83.5	37.2
	Specific	98.3	88.6	98.1	62.8

- Simulation results for 750 assets:

Risk Model	Risk	Stock Selection		Factor Tilt	
		Forecast over Truth (%)	**Ratio of Component to Active Variance (%)**	**Forecast over Truth (%)**	**Ratio of Component to Active Variance (%)**
Factor Based	**Active Variance**	97.2	100.0	80.9	100.0
	Factor	65.5	2.8	65.4	53.5
	Specific	98.1	97.2	98.2	46.5

Adding Constraints

- So far, we have been looking at unconstrained optimizations
- What if there are constraints?
  - Conventional wisdom: constraints act to limit the error-maximizing behavior of optimization
- Consider the case in which a manager constrains *J* characteristics of an (active) portfolio with *N* assets to be exactly zero by imposing the constraints:

$$Ah = 0$$

Adding Constraints

- These equality constraints effectively reduce the number of variables in the problem, since they enable us to write the optimization problem in terms of *N-J* assets, rather than *N*, as follows:

$$Ah = A_{J\times J}h_{J\times 1} + A_{J\times(N-J)}h_{(N-J)\times 1} = 0$$

$$h_{J\times 1} = -\underbrace{A_{J\times J}^{-1}A_{J\times(N-J)}}_{Q_{J\times(N-J)}}h_{(N-J)\times 1}$$

- In turn, this generally reduces the forecasting bias
  - Since factor risk is $w'Fw$
  - When we constrain a factor, say *i*, we set $w_i = 0.$
  - Effectively drops a variable from the problem
  - Moreover, drops it from the factor risk, which is the principal source of forecasting bias

Adding Constraints: Simulations

- Rerun simulations:
  - Case 1: Constrain all factor exposures to be zero, except for the three factors comprising the alpha
  - Case 2: Add long-only constraint

Risk Model	Risk	Stock Selection		Factor Tilt	
		Forecast over Truth (%)	**Ratio of Component to Active Variance (%)**	**Forecast over Truth (%)**	**Ratio of Component to Active Variance (%)**
Factor Neutral	**Active Variance**	98.2	100.0	95.6	100.0
	Factor	--	0.0	93.2	54.2
	Specific	98.2	100.0	98.2	45.8
Long only Active Risk 3%	**Active Variance**	95.9	100.0	89.3	100.0
	Factor	84.4	19.7	81.1	52.5
	Specific	98.7	80.3	98.5	47.5

Conclusion

- Due to noise in the covariance matrix, portfolio optimization tends to produce portfolios for which the risk forecasts are underestimates of the true risk
- In the case in which the asset returns have a factor structure, using a factor-based covariance matrix mitigates the risk forecast bias significantly
- Furthermore, our analysis reveals that the bias in factor model risk forecasts may be significantly less than earlier estimates would suggest
- Finally, we discuss briefly how constraints mitigate the forecast bias

References

- Presentation is based on the paper: "Forecast Risk Bias in Optimized Portfolios", MSCI Barra Research Insight, October 2009 – Bender, Lee, Stefek, Yao
- Additional citations:

- Barry, C. (1974), “Portfolio Analysis Under Uncertain Means, Variances, and Covariances”, Journal of Finance.
- Jorion, P. (1992), “Portfolio Optimization in Practice,” Financial Analyst Journal.
- Kritzman, M. (2006), “Are Optimizers Error Maximizers?” Journal of Portfolio Management, Summer 2006.
- Ledoit, O. and M. Wolf (2004), “Honey I Shrunk the Sample Covariance Matrix” Journal of Portfolio Management.
- Michaud, R. (1989), “The Markowitz optimization enigma: Is optimized optimal?” Financial Analysts Journal.

MSCI 24 Hour Global Client Service

Americas	
Americas	1.888.588.4567 (toll free)
Atlanta	+1.404.551.3212
Boston	+1.617.532.0920
Chicago	+1.312.706.4999
Monterrey	+52.81.1253.4020
Montreal	+1.514.847.7506
New York	+1.212.804.3901
San Francisco	+1.415.836.8800
São Paulo	+55.11.3706.1360
Stamford	+1.203.325.5630
Toronto	+1.416.628.1007

Europe, Middle East & Africa	
Amsterdam	+31.20.462.1382
Cape Town	+27.21.673.0100
Frankfurt	+49.69.133.859.00
Geneva	+41.22.817.9777
London	+44.20.7618.2222
Madrid	+34.91.700.7275
Milan	+39.02.5849.0415
Paris	0800.91.59.17 (toll free)
Zurich	+41.44.220.9300

Asia Pacific	
China North	10800.852.1032 (toll free)
China South	10800.152.1032 (toll free)
Hong Kong	+852.2844.9333
Seoul	+827.07688.8984
Singapore	800.852.3749 (toll free)
Sydney	+61.2.9033.9333
Tokyo	+81.3.5226.8222

clientservice@msci.com

www.msci.com | www.riskmetrics.com

Barra Knowledge Base – Online Answers to Barra Questions: www.barra.com/support

Notice and Disclaimer

- This document and all of the information contained in it, including without limitation all text, data, graphs, charts (collectively, the “Information”) is the property of MSCI Inc. or its subsidiaries (collectively, “MSCI”), or MSCI’s licensors, direct or indirect suppliers or any third party involved in making or compiling any Information (collectively, with MSCI, the “Information Providers”) and is provided for informational purposes only. The Information may not be reproduced or redisseminated in whole or in part without prior written permission from MSCI.
- The Information may not be used to create derivative works or to verify or correct other data or information. For example (but without limitation), the Information many not be used to create indices, databases, risk models, analytics, software, or in connection with the issuing, offering, sponsoring, managing or marketing of any securities, portfolios, financial products or other investment vehicles utilizing or based on, linked to, tracking or otherwise derived from the Information or any other MSCI data, information, products or services.
- The user of the Information assumes the entire risk of any use it may make or permit to be made of the Information. NONE OF THE INFORMATION PROVDERS MAKES ANY EXPRESS OR IMPLIED WARRANTIES OR REPRESENTATIONS WITH RESPECT TO THE INFORMATION (OR THE RESULTS TO BE OBTAINED BY THE USE THEREOF), AND TO THE MAXIMUM EXTENT PERMITTED BY APPLICABLE LAW, EACH INFORMATION PROVIDER EXPRESSLY DISCLAIMS ALL IMPLIED WARRANTIES (INCLUDING, WITHOUT LIMITATION, ANY IMPLIED WARRANTIES OF ORIGINALITY, ACCURACY, TIMELINESS, NON-INFRINGEMENT, COMPLETENESS, MERCHANTABILITY AND FITNESS FOR A PARTICULAR PURPOSE) WITH RESPECT TO ANY OF THE INFORMATION.
- Without limiting any of the foregoing and to the maximum extent permitted by applicable law, in no event shall any Information Provider have any liability regarding any of the Information for any direct, indirect, special, punitive, consequential (including lost profits) or any other damages even if notified of the possibility of such damages. The foregoing shall not exclude or limit any liability that may not by applicable law be excluded or limited, including without limitation (as applicable), any liability for death or personal injury to the extent that such injury results from the negligence or wilful default of itself, its servants, agents or sub-contractors.
- Information containing any historical information, data or analysis should not be taken as an indication or guarantee of any future performance, analysis, forecast or prediction. Past performance does not guarantee future results.
- None of the Information constitutes an offer to sell (or a solicitation of an offer to buy), any security, financial product or other investment vehicle or any trading strategy.
- MSCI’s indirect wholly-owned subsidiary Institutional Shareholder Services, Inc. (“ISS”) is a Registered Investment Adviser under the Investment Advisers Act of 1940. Except with respect to any applicable products or services from ISS (including applicable products or services from MSCI ESG Research Information, which are provided by ISS), none of MSCI’s products or services recommends, endorses, approves or otherwise expresses any opinion regarding any issuer, securities, financial products or instruments or trading strategies and none of MSCI’s products or services is intended to constitute investment advice or a recommendation to make (or refrain from making) any kind of investment decision and may not be relied on as such.
- The MSCI ESG Indices use ratings and other data, analysis and information from MSCI ESG Research. MSCI ESG Research is produced by ISS or its subsidiaries. Issuers mentioned or included in any MSCI ESG Research materials may be a client of MSCI, ISS, or another MSCI subsidiary, or the parent of, or affiliated with, a client of MSCI, ISS, or another MSCI subsidiary, including ISS Corporate Services, Inc., which provides tools and services to issuers. MSCI ESG Research materials, including materials utilized in any MSCI ESG Indices or other products, have not been submitted to, nor received approval from, the United States Securities and Exchange Commission or any other regulatory body.
- Any use of or access to products, services or information of MSCI requires a license from MSCI. MSCI, Barra, RiskMetrics, ISS, CFRA, FEA, and other MSCI brands and product names are the trademarks, service marks, or registered trademarks or service marks of MSCI or its subsidiaries in the United States and other jurisdictions. The Global Industry Classification Standard (GICS) was developed by and is the exclusive property of MSCI and Standard & Poor’s. “Global Industry Classification Standard (GICS)” is a service mark of MSCI and Standard & Poor’s.